

5.1 Differentiation

Question Paper

Course	CIEAS Maths
Section	5. Differentiation
Topic	5.1 Differentiation
Difficulty	Hard

Time allowed: 70
Score: /51
Percentage: /100

Question 1a

For each of the following, find $\frac{dy}{dx}$ in terms of x :

(a) $y = -3x^3 + 5x^2 - 3x + \sqrt{13}$

[2 marks]

Question 1b

(b) $y = 9x^{\frac{1}{3}} - 6x^{-\frac{1}{3}}$

[2 marks]

Question 2

Given that $y = \frac{1}{\sqrt{x}} \left(1 + \frac{1}{x}\right)$, $x > 0$, find $\frac{dy}{dx}$.

[3 marks]

Question 3a

For each of the following, find $\frac{dy}{dx}$ in terms of x :

(a) $y = (2x - 1)^2(x + 1)$

[3 marks]

Question 3b

(b) $y = \frac{1}{x^5}(x^2 + \sqrt{x} - 1)$

[3 marks]

Question 4a

For each of the following, use the chain rule to find $\frac{dy}{dx}$ in terms of x :

(a) $y = \left(\frac{1}{x} + 2x\right)^4$

[4 marks]

Question 4b

$$(b) y = \frac{1}{(x^3-1)^2}$$

[4 marks]

Question 5

The function f is defined by $f(x) = x^3 - 4x^2 + 6x - 9$. Show that there are no solutions to the equation $f'(x) = 0$.

[4 marks]

Question 6a

A curve has the equation $y = \frac{3}{8}x^{\frac{4}{3}} - 12x^{\frac{1}{3}}$.

(a) Show that $\frac{dy}{dx} = ax^{-\frac{2}{3}}(x + b)$, where a and b are rational numbers to be found.

[3 marks]

Question 6b

(b) Hence find the coordinates of the point on the curve where the gradient is 0.

[2 marks]

Question 7a

Given that $y = (x^2 - 2x - 3)^4$

(a) use the chain rule to find $\frac{dy}{dx}$

[3 marks]

Question 7b

(b) find the coordinates of any stationary points and determine their nature

[4 marks]

Question 7c

(c) sketch the curve.

[3 marks]

Question 8

A curve has the equation $y = 4x^3 + bx^2 + 3x - 17$, where b is a constant. Given that there is only one point on the curve where the gradient is zero, determine the possible values of b .

[4 marks]

Question 9

A curve is described by the equation $4y^2 - 3x^5 = 0$, $y > 0$.

By rearranging the equation to make y the subject, find $\frac{dy}{dx}$.

[2 marks]

Question 10

The curve with equation $y = ax^2 + bx + c$ has a gradient of 8 at the point $(-2, 0)$, and a gradient of -10 at the point $(1, -3)$. Find the values of a , b and c .

[5 marks]

